Pergamon

J. Appl. Maths Mechs, Vol. 62, No. 3, pp. 477–478, 1998

0021–8928/98/\$—see front matter

PII: S0021–8928(98)00060-4

THE PROBLEM OF AN EARTH DAM†

P. Ya. KOCHINA and N. N. KOCHINA

Moscow

(*Received* 26 *March* 1997)

The behaviour of the solution of the problem of a rectangular earth dam in the neighbourhood of a singular point at the intersection of the free surface and the seepage area is investigated. A similar result is obtained for an earth dam with a slanting lower incline.

The solution of the problem of seepage across a rectangular earth dam, found by P. Ya. Kochina (P. Ya. Polubarinova-Kochina) and studied in detail in [1, 2], has been investigated in even greater detail with a description of six limiting cases, in each of which one or certain constant dimensionless characteristics determining the solution of the actual problem vanishes or becomes infinite [3, Table 2 on p. 76].

For rectangular earth dam (Fig. 1), in the general case when the dimensionless parameters a and b, on which the solution of the problem depends, satisfy the inequalities

$$1 < a < b < \infty \tag{1}$$

in the domains determining the motion, there are five singular points: A, B, C, D and E. In each of the above-mentioned six limiting cases, the strict inequalities (1) are replaced by some of the following inequalities with equalities

$$1 \leqslant a \leqslant b \leqslant \infty \tag{2}$$

We will investigate the behaviour of certain required functions of the problem in the neighbourhood of the singular point A in the boundary between the free surface and the seepage area (which is the same for the general case and all the limiting cases).

In the plane $w = d\omega/dz = u - i\upsilon$, the solution of the dam problem corresponds to Fig. 2. Here, $z = xi + iy$, $\omega = \varphi + i\psi$, φ is the velocity potential, ψ is the stream function, w is a complex velocity, ω is a complex potential, u and υ are the velocity components of the seepage along the x and y axes and $\varkappa$ is the seepage coefficient.

The point A in Figs 1 and 2 separates the free surface BA from the seepage area AE. It can be seen from Fig. 2 that the condition

$$u^2 + \upsilon^2 + \varkappa\upsilon = 0 \tag{3}$$

is satisfied on the free surface and that the condition

$$\upsilon = -\varkappa \tag{4}$$

is satisfied in the seepage area AE. The values of the parameters a and b (by virtue of (1) and (2), $a \leqslant b$) correspond to the singular points C and D in the auxiliary complex plane (Fig. 3). The domain of the complex potential is shown in Fig. 4.

It is clear from Fig. 2 and formula (3) that the equalities $u = 0$, $\upsilon = -\varkappa$ hold at the point A.

In the ζ plane, the segment $0 \leqslant \zeta \leqslant 1$ corresponds to the free surface and the seepage area is the ray $\zeta \leqslant 0$. Here, the value $\zeta = 0$ corresponds to point A (Fig. 3).

It follows from the solution of the problem of a rectangular earth dam [1, 2] that

$$dy/2x = -K(1-\zeta)/K(\zeta) \quad (0 < \zeta < 1) \tag{5}$$

Here, $K(\zeta)$ is the complete elliptic integral of the first kind, which is considered as a function of the square of the modulus $k^2 = \zeta$. It follows from formula (5) that $dy/dx = -\infty$ when $\zeta \to 0$, that is, in Fig. 1, the tangent to the free surface at point A is vertical, since $dy/dx = \pi^{-1} \ln \zeta$.

Suppose that $x = l$, $y = y_0$ at point A. Using formula (5), we find the asymptotic representation

$$y \approx y_0 - \pi^{-1}(l-x)\ln(l-x) \tag{6}$$

close to the value $x = l$.

†*Prikl. Mat. Mekh.* Vol. 62, No. 3, pp. 515–516, 1998.

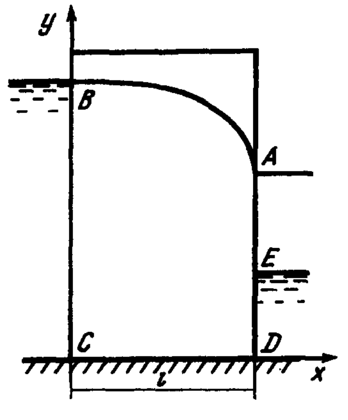

Fig. 1.

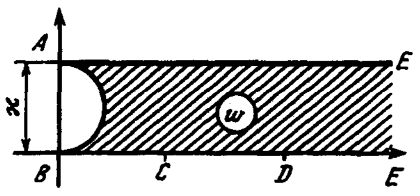

Fig. 2.

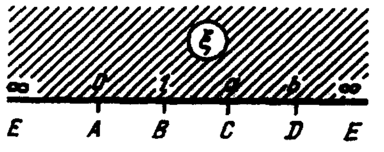

Fig. 3.

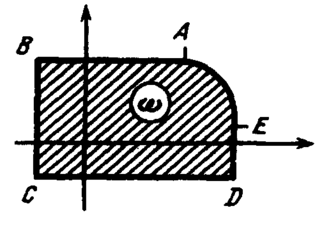

Fig. 4.

Such is the behaviour, in the neighbourhood of point A, of the solution of the problem of a rectangular dam both for the general case as well as for all the remaining cases.

In Fig. 4 on the arc EA, the parametric dependence of ψ on φ is given by the formulae (G is a certain constant with the dimension of length)

$$\varphi = \varphi_0 - \varkappa G \int \frac{K(1/(1-\zeta))d\zeta}{(1-\zeta)\sqrt{(a-\zeta)(b-\zeta)}}, \quad \psi = \psi_0 + \varkappa G \int \frac{K(\zeta/(\zeta-1))d\zeta}{(1-\zeta)\sqrt{(a-\zeta)(b-\zeta)}}$$

It is seen that $d\psi/d\varphi = 0$ at point A and that $d\varphi/d\psi = 0$ at point E.

In the case of an earth dam with a slanting bottom incline (at an angle $\pi\alpha$, $1/2 < \alpha < 1$), the equation of which is $y = x \operatorname{tg} \alpha$, the equation of the free surface BA in the domain w again has the form (3) and, in the seepage area AE, the relation

$$u \cos \alpha + (v + \varkappa) \sin \alpha = 0 \tag{7}$$

is obtained instead of (4).

Solving (3) and (7) simultaneously, we obtain $v/u = \operatorname{tg} \alpha$, that is, in this case also the free surface is in contact with the seepage area at a point corresponding to point A.

REFERENCES

1. POLUBARINOVA-KOCHINA, P. Ya., *Theory of Ground Water Motion*. Nauka, Moscow, 1977.
2. POLUBARINOVA-KOCHINA, P. Ya., *Hydromechanics of Subterranean Waters and Problems of Irrigation*. Nauka, Moscow, 1994.
3. POLUBARINOVA-KOCHINA, P. Ya., *Problems of Motions with a Free Surface in Subterranean Hydrodynamics*. Editorial Board of the Journal *Uspekhi Fiz. Nauk*, Moscow, 1996.

Translated by E.L.S.